Name: ______________

TEACHER: ______________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2014

Year 12

H S C COURSE

ASSESSMENT TASK 2

Half Yearly Examination

Extension 1 Mathematics

Examiners ~P.Biczo, S. Faulds, D. Crancher, G. Rawson, S. Gee

GENERAL INSTRUCTIONS

- Reading time – 5 minutes.
- Working time – 90 minutes.
- Attempt all questions.
- Board approved calculators and board approved mathematical templates may be used.
- This examination paper must NOT be removed from the examination room.
- Section A consists of five (5) multiple choice questions worth 1 mark each. Fill in your answer on the multiple choice answer sheet provided.
- Section B requires all necessary working to be shown in every question. This section consists of five (5) questions worth nine (9) marks each.
- Marks may not be awarded for careless or badly arranged work.
- **Start each question in a new answer booklet.** Write your student number on every sheet.
- Additional booklets are available if required.

Section A: 5 Multiple Choice Questions (1 mark each)

Use the multiple choice answer sheet, provided on the back of the exam paper, to answer questions 1 – 5. The answer sheet may be torn off.

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1. Consider the polynomial $P(x) = 2x^3 + x^2 + 2x + a$. If $x - 1$ is a factor of $P(x)$, the value of a is ?

A. –6 **B.** –5 **C.** 5 **D.** 6

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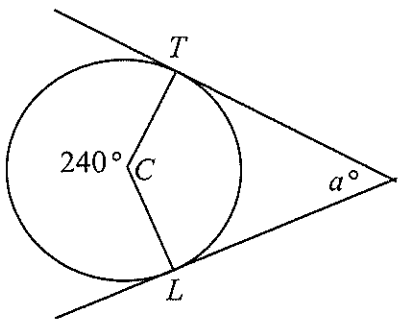
2. A curve has parametric equations $x = t + 1$ and $y = 2t^2$. What is Cartesian equation of this curve?

A. $y = 2\sqrt{x-1}$ **B.** $y = 2\sqrt{x+1}$ **C.** $y = 2(x+1)^2$ **D.** $y = 2(x-1)^2$

3.



T and L are the points of contact of the tangents to the circle, with centre C and reflex $\angle TCL = 240°$.

NOT TO SCALE

The value of a is:

A. 30° **B.** 60° **C.** 120° **D.** unable to be determined as insufficient information

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4. ${}^{n}P_{12}$ is equivalent to:

A. $\frac{n!}{12!}$ **B.** $\frac{n!}{(n-12)!}$ **C.** $\frac{(n-12)!}{n!}$ **D.** $\frac{n!}{12!(n-12)!}$

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5. The solution to the equation $\sin 2x - \cos x = 0$ for $0 \le x \le 2\pi$ is

A. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **B.** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$ **C.** $\frac{\pi}{6}, \frac{5\pi}{6}$ **D.** $\frac{\pi}{6}, \frac{7\pi}{6}$

Section B : 5 Questions (9 marks each)

Show all necessary working. Commence a new answer sheet for each question.

Question 6 : **Commence a new answer sheet** **Marks**
Write your student number/name on the answer sheet.

a) It is given that the polynomial $f(x) = x^3 - x^2 - x - 1$ has a zero between $x = 1$ and $x = 2$.

(i) Taking $x = 2$ as a first approximation to this zero, use Newton's method to calculate a second approximation. **2**

(ii) Give a geometrical interpretation of the process used in part (i). **1**

b) The polynomial $A(x) = (x-a)^3 + b$ has a zero when $x = 1$ and, when divided by x, the remainder is -7. Find all possible values of a and b. **2**

c) In each of the following parts, use the information to obtain the required real polynomial $P(x)$ explicitly in the form $P(x) = p_0x^n + p_1x^{n-1} + p_2x^{n-2} + ... + p_n$ where $n, p_0, p_1, ..., p_n$ are to be given ***numerical*** values.

(i) $P(x)$ is quadratic, $P(0) = 15$, and the minimum value of $P(x)$ is 3 when $x = 2$. **2**

(ii) $P(x)$ is an even polynomial function of degree 4, has $(x+2)^2$ as a factor and has a remainder 50 on division by $x - 3$. **2**

Question 7 : Commence a new answer sheet **Marks**
Write your student number/name on the answer sheet.

a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.
The tangent at P and the line through Q parallel to the y-axis intersect at the point T.

(i) Show that the equation of the tangent at P is $y = px - ap^2$. **2**

(ii) Find the coordinates of T **1**

(iii) Write down the coordinates of M, the midpoint of PT **1**

(iv) Determine the locus of M when $pq = -1$. **1**

b) Two points P and Q move on the parabola $x^2 = 4ay$ so that their x values differ by a.

(i) P has coordinates (x_1, y_1). Show that the midpoint C, of PQ is

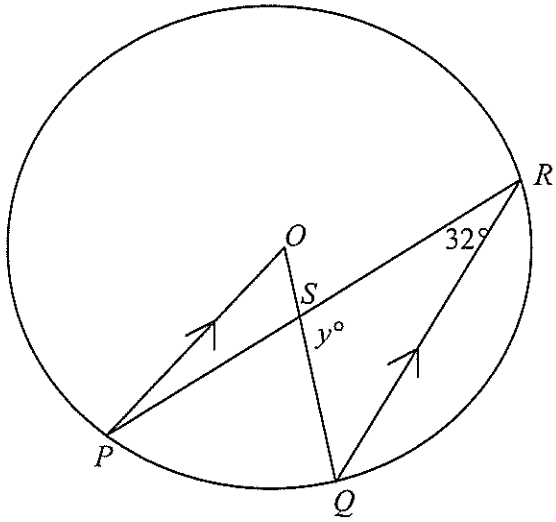
$$\left(\frac{2x_1 + a}{2}, \frac{2x_1^2 + 2ax_1 + a^2}{8a}\right)$$ **2**

(ii) Show that the locus of C is another parabola. **2**

Question 8 : **Commence a new answer sheet** **Marks**
Write your student number/name on the answer sheet.

a) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3, for all positive integers n. **3**

b)



NOT TO SCALE

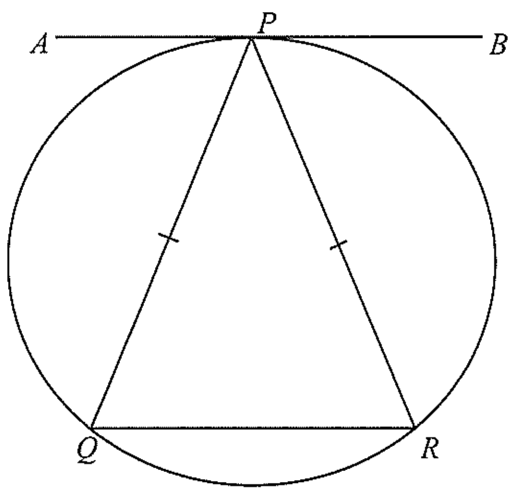
The points P, Q, and R lie on a circle with centre O.
The lines PO and QR are parallel, and OQ and PR intersect at S.
Also, $\angle QSR = y°$ and $\angle PRQ = 32°$, as shown in the diagram.

Copy or trace the diagram into your writing booklet.

(i) State why $\angle POQ = 64°$. **1**

(ii) Find y. Justify your answer with appropriate geometrical reasoning. **2**

c)



NOT TO SCALE

Given that $PQ = PR$ and AB is a tangent to circle PQR at P, prove AB is parallel to QR. **3**

Question 9 : Commence a new answer sheet **Marks**

Write your student number/name on the answer sheet.

a) Eight friends arrive at a restaurant for dinner. How many different arrangements are possible if Sue and Ray are to be seated together and Kerrie and Max are to be seated together, around a circular table? **1**

b) Mr. Seblani is required to allocate leadership awards to SRC representatives from Years 12, 11 and 10. The recipients will be ranked in order of merit (1^{st}, 2^{nd}, 3^{rd} , etc.) and there are to be at most 3 Year 12 recipients, at most 2 Year 11 recipients and at most 1 Year 10 recipient.

(i) Find the number of different ways that Mr. Seblani can distribute ~~the~~ 6 awards across the three Year groups. (Note: When Mr. Seblani allocates the award, he is only interested in the Year group that the representative belongs to, not who the representative is.) **2**

(ii) Mr. Seblani decides that only 5 awards will be required. How many different allocations across the three Year groups are possible if the same selection criteria is used? **2**

c) Sets of playing cards are created with a single letter of the alphabet printed on each card, giving 26 cards in a deck. There are 6 decks of these cards, each of a different colour. If all of these cards are combined and shuffled:

(i) How many different hands of 4 cards are possible? **1**

(ii) How many of these hands will contain 3 B's? **1**

(iii) How many of the hands will contain at least 3 cards from the same deck? **2**

Question 10 : **Commence a new answer sheet** **Marks**

Write your student number/name on the answer sheet.

a) The point $(5, 5)$ divides the interval between $(-1, 4.6)$ and $(q, 6)$ in the ratio 2 : 5. **1**
Find the value of q.

b) Solve for x: $\frac{3}{4-x} > 1$ **2**

c) The curves $y = (x-2)^2$ and $y = x^2$ intersect at the point $(1, 1)$. Find, correct to the nearest degree, the acute angle between the curves at their point of intersection. **2**

d) Express $\cos 4x$ as a polynomial in $\cos x$. Justify your working. **2**

e) Show that if $0 < x < \frac{\pi}{4}$, then $\sin 2x > 2\sin^2 x$. **2**
Justify your working.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

NAME: ________________________________

TEACHER: ____________________________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2014 Year 12 Mathematics Extension Task 2

Section A - Answer Sheet

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ

NOTE : $\ln x = \log_e x, \quad x > 0$

NAME: ANSWERS

TEACHER: DILLON

HURLSTONE AGRICULTURAL HIGH SCHOOL

2014 Year 12 Mathematics Extension Task 2

Section A - Answer Sheet

1	A	**B**	C	D
2	A	B	C	**D**
3	A	**B**	C	D
4	A	**B**	C	D
5	**A**	B	C	D

Year 12	Mathematics Extension 1 Half Yearly Examination	Task 2 HSC 2014
Question No.6	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

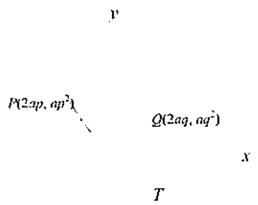
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

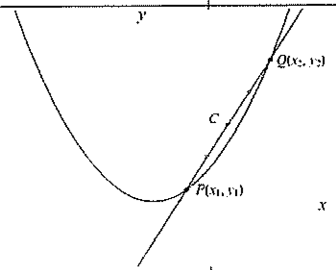
Outcome	Solutions	Marking Guidelines
	a.(i) $f(x)=x^3-x^2-x-1$ $f'(x)=3x^2-2x-1$ $f(2)=(2)^3-(2)^2-2-1=1$ $f'(2)=3\times(2)^2-2\times(2)-1=7$ $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ $x_2=2-\frac{1}{7}=\frac{13}{7}$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
	(ii)Newton's method uses the tangent's, at the point on the curve for the initial approximation, intercept with the x axis as the next approximation to the root.	1 mark correct answer
	b. $A(x)=(x-a)^3+b$ $A(1)=0,\ A(0)=-7$ $(1-a)^3+b=0$ $1-3a+3a^2-a^3+b=0$ $-a^3+b=-7$ $b=a^3-7$ $\therefore 1-3a+3a^2-a^3+a^3-7=0$ $-6-3a+3a^2=0$ $3(a^2-a-2)=0$ $3(a-2)(a+1)=0$ $a=2,b=1$ $a=-1,b=-8$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
	c.(i) $P(x)=ax^2+bx+c$ $P'(x)=2ax+b$ $P(0)=c=15$ $P'(2)=2a(2)+b=0$ $P(2)=a(2)^2+b(2)+c=3$ $4a+b=0$ $4a+2b+15=3$ $b=-12$ $a=3$ $P(x)=3x^2-12x+15$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
	(ii) $P(x)=a(x+2)^2(x-2)^2$ $P(3)=50$ $a(3+2)^2(3-2)^2=50$ $a=2$ $P(x)=2(x+2)^2(x-2)^2$ $P(x)=2[(x-2)(x+2)]^2=2[x^2-4]^2$ $P(x)=2x^4-16x^2+32$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.

Year 12 Task 2 2014 – Extension 1 Mathematics

Question No: 7 **Solutions and Marking Guidelines**

PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Sample Solution	Marking Guidelines
PE3	(a)(i) $x^2 = 4ay$ $y = \frac{x^2}{4ay}$ $\frac{dy}{dx} = \frac{x}{2a}$ $m = \frac{2ap}{2a} = p$ at $x = 2ap$  So equation of tangent is $y - y_1 = m(x - x_1)$ $y - ap^2 = p(x - 2ap)$ $y = px - 2ap^2 + ap^2$ $y = px - ap^2$	**2 marks** – correct solution **1 mark** – substantially correct solution
	(ii) line through Q is $x = 2aq$ Sub into tangent $y = px - ap^2$ $= p(2aq) - ap^2$ $= 2apq - ap^2$ $\therefore T$ is $(2aq, 2apq - ap^2)$	**1 mark** – correct solution
	(iii) midpoint of PT is: $M\left(\frac{2ap + 2aq}{2}, \frac{2apq - ap^2 + ap^2}{2}\right)$ $= (a(p+q), apq)$	**1 mark** – correct solution
	(iv) when $pq = -1$, $y = apq$ $= -a$ ie the locus of M is the line $y = -a$.	**1 mark** – correct solution

Outcome	Sample Solution	Marking Guidelines
	(b)(i) P is (x_1, y_1), let Q be (x_2, y_2) Q has $x_2 = x_1 + a$ sub into parabola to find y_2...  $x^2 = 4ay$ $(x_1 + a)^2 = 4ay_1$ $y_1 = \frac{(x_1 + a)^2}{4a} = \frac{x_1^2 + 2x_1a + a^2}{4a}$ Midpoint, C, of PQ is... $C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $= \left(\frac{x_1 + x_1 + a}{2}, \frac{\frac{x_1^2}{4a} + \frac{x_1^2 + 2ax_1 + a^2}{4a}}{2}\right)$ $= \left(\frac{2x_1 + a}{2}, \frac{2x_1^2 + 2ax_1 + a^2}{8a}\right)$	**2 marks** – correct solution **1 mark** – substantial progress towards correct solution
	(ii) $x = \frac{2x_1 + a}{2} \Rightarrow x_1 = \frac{2x - a}{2}$ ...[1] $y = \frac{2x_1^2 + 2ax_1 + a^2}{8a}$ ...[2] $= \frac{2\left(\frac{2x-a}{2}\right)^2 + 2a\left(\frac{2x-a}{2}\right) + a^2}{8a}$ (sub[1] → [2]) $= \frac{\frac{2(4x^2 - 4ax + a^2)}{4} + \frac{4ax - 2a^2}{2} + a^2}{8a}$ $= \frac{4x^2 - 4ax + a^2 + 4ax - 2a^2 + 2a^2}{16a}$ $y = \frac{4x^2 + a^2}{16a}$, which is a quadratic $\therefore$ the locus of C is a parabola.	**2 marks** – substantially correct solution **1 mark** – substantial progress towards correct solution

Year 12 2014	Mathematics Extension 1	Task 2 Half Yearly

Solutions and Marking Guidelines

Outcome Addressed in this Question

HE2 - uses inductive reasoning in the construction of proofs
PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

	Solutions	Marking Guidelines
	Question 8	
HE3	a) Prove true for $n = 1$ $(1)^3+2(1) = 1+2 = =3$ which is divisible by 3 Therefore true for $n = 1$ Assume true for $n = k$, i.e. $k^3+2k = 3M$ where M is any integer. Prove true for $n = k+1$, $(k+1)^3+2(k+1)$ $= k^3+3k^2+3k+1+2k+2$ $= k^3+2k+3k^2+3k+3$ $= 3M+3(k^2+k+1)$ since $k^3+2k = 3M$ where M is any integer $= 3(M+k^2+k+1)$ Which is divisible by 3 Therefore, true for $n = k+1$. Thus, if the result is true for $n = k$, it is true for $n = k+1$. It has been shown that it is true for $n = 1$, hence it is true for $n = 2$. It is therefore correct for $n = 3$ and so on for all positive integers n.	**3 marks for complete correct solution** **2 marks for substantial progress to correct solution** **1 mark for limited progress to correct solution**
PE3	b) i) $\angle POQ = 64°$ since the angle at the centre is twice the angle at the circumference standing on the same arc.	**1 mark for complete correct reasoning**
PE3	ii) $\angle OQR = \angle POQ$ (Alternate angles are equal, $PO // QR$) $= 64°$ $\angle RSQ + \angle SRQ + \angle RQS = 180°$ (Angle sum of ΔSQR is 180°) $\therefore y° + 32° + 64° = 180°$ $\therefore y° = 84°$	**2 marks for complete correct solution with correct reasoning** **1 mark for substantial progress to correct solution**
PE3	iii) $\angle BPR = \angle PQR$ (Alternate segment theorem) $\angle PRQ = \angle PQR$ (Angles opposite equal sides in an isosceles triangle) $\therefore \angle BPR = \angle PRQ$ $\therefore AB // QR$ since alternate angles are equal	**3 marks for complete correct solution with correct reasoning** **2 marks for substantial progress to correct solution** **1 mark for limited progress to correct solution**

Year 12 Mathematics Extension 1 Half Yearly Examination 2014		
Question No. 9	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
PE3	**solves problems involving permutations and combinations**, inequalities, polynomials, circle geometry and parametric representations	
Outcome	**Solutions**	**Marking Guidelines**
PE3	**(a)** 2 couples and 4 others arranged around a round table, treat as 6 "items". No. of arrangements $= (6-1)! \times 2! \times 2!$ $= 480$	**1 mark** Correct solution.
PE3	**(b) (i)** 6 awards $\rightarrow 3 \times$ Yr 12, $2 \times$ Yr 11, $1 \times$ Yr 10 No. of ways $= \dfrac{6!}{3!2!}$ $= 60$	**2 marks** Correct solution. **1 mark** Correctly determines allocation of the six awards.
PE3	**(ii)** 5 awards $\rightarrow 3 \times$ Yr 12, $1 \times$ Yr 11, $1 \times$ Yr 10 or $3 \times$ Yr 12, $2 \times$ Yr 11 or $2 \times$ Yr 12, $2 \times$ Yr 11, $1 \times$ Yr 10 No. of ways $= \dfrac{5!}{3!} + \dfrac{5!}{3!2!} + \dfrac{5!}{2!2!}$ (Yr 12 and Yr 11 students are "like items".) $= 20 + 10 + 30$ $= 60$	**2 marks** Correct solution. **1 marks** Substantial progress towards correct solution, showing possible allocations of the 5 awards.
PE3	**(c) (i)** No. of hands $= {}^{156}C_4$ $= 23\ 738\ 715$ (numerical answer not necessary)	**1 mark** Correct answer.
PE3	**(ii)** Hands containing 3 'B's $\rightarrow$ 3 out of 6 'B's chosen, remaining card to be chosen from $(156-6)$ cards No. of hands $= {}^{6}C_3 \times 150$ $= 3\ 000$ (numerical answer not necessary)	**1 mark** Correct solution.
PE3	**(iii)** Hands containing at least 3 cards from the same deck $\rightarrow$ 6 decks. 3 cards chosen from 1 deck + 1 from remaining 130 cards or 4 cards from 1 deck. No. of hands $= 6 \times {}^{26}C_3 \times 130 + 6 \times {}^{26}C_4$ $= 2\ 028\ 000 + 89\ 700$ $= 2\ 117\ 700$ (numerical answer not necessary)	**2 marks** Correct solution. **1 mark** Substantial progress towards full solution.

Outcomes Addressed in this Question

PE3 **solves problems involving inequalities**

HE7 **evaluates mathematical solutions to problems and communicates them in an appropriate form.**

a) At point of division, $x = \dfrac{mx_2 + nx_1}{m+n}$.

$\therefore 5 = \dfrac{2\times q + 5\times -1}{2+5}$.

$\therefore 35 = 2q - 5$

$\therefore q = 20.$

1 mark : correct answer

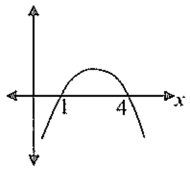
b) $\dfrac{3}{4-x} > 1$

$\dfrac{3(4-x)^2}{4-x} > (4-x)^2$

$3(4-x) - (4-x)^2 > 0$

$(4-x)(3-(4-x)) > 0$

$(4-x)(x-1) > 0$



From the graph, $1 < x < 4$.

2 marks: correct solution
1 mark : significant progress towards correct solution

c) For $y = (x-2)^2$, $\dfrac{dy}{dx} = 2(x-2)$ and when $x = 1$, $\dfrac{dy}{dx} = -2$.

For $y = x^2$, $\dfrac{dy}{dx} = 2x$. and when $x = 1$, $\dfrac{dy}{dx} = 2$.

Let $m_1 = -2$ and $m_2 = 2$.

Angle θ between the curves is $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \tan\theta = \left|\dfrac{-2-2}{1-2\times 2}\right| = \dfrac{4}{3}$

$\therefore \theta = 53°$ (nearest degree)

2 marks : correct solution
1 mark : significant progress towards correct solution

d) $\cos 4x = \cos(2\times 2x)$

$= 2\cos^2(2x) - 1$

$= 2(\cos(2x))^2 - 1$

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1.$

2 marks : correct solution
1 mark : significant progress towards correct solution

e) Consider the difference $\sin 2x - 2\sin^2 x$

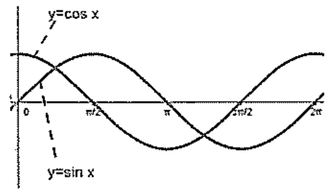
$= 2\sin x\cos x - 2\sin^2 x$

$= 2\sin x(\cos x - \sin x)$

Given $0 < x < \dfrac{\pi}{4}$, $\sin x$ is positive.

Also, for $0 < x < \dfrac{\pi}{4}$, $\cos x > \sin x$, as can be seen from the graph

(graphs meet at $x = \dfrac{\pi}{4}$)



$\therefore 2\sin x(\cos x - \sin x) = 2 \times$ positive $\times$ positive

$=$ positive

$\therefore \sin 2x - 2\sin^2 x > 0$

$\therefore \sin 2x > 2\sin^2 x.$

2 marks : correct solution, with justification
1 mark : significant progress towards correct solution